4-1 Videos Guide

4-1a

- Form of a power series
  - $\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$

Exercise:

- Find the radius and interval of convergence of $\sum_{n=0}^{\infty}(x+2)^n$.

Theorem (statement):

- For a given power series $\sum_{n=0}^{\infty} a_n (x-a)^n$, there are only three possibilities:
  (i) The series converges only for $x = a$
  $R = 0$; interval: $\{a\}$
  (ii) The series converges for all $x \in \mathbb{R}$
  $R = \infty$; interval: $(-\infty, \infty)$
  (iii) There is a number $R > 0$ such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$
  Note: Convergence at the endpoints of the interval is determined by testing them individually.

Exercises:

Find the radius of convergence and interval of convergence of the series.

4-1b

- $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$
- $\sum_{n=1}^{\infty} n^n x^n$
- $\sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$

4-1c

- $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n5^n} x^n$
- $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n} (x-1)^n$

4-1d

$\sum_{n=2}^{\infty} \frac{b^n}{\ln n} (x-a)^n, \quad b > 0$

4-1e

- $\sum_{n=1}^{\infty} \frac{n! x^n}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}$